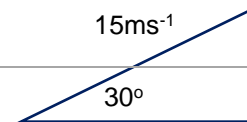


# Projectile Questions (Answers) for A-level Physics

## Basic Concepts

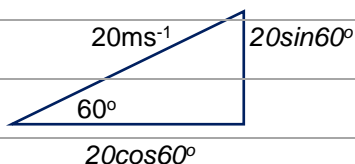
1. A stone is thrown at an angle of  $30^\circ$  with an initial velocity of  $15 \text{ ms}^{-1}$ . Calculate the horizontal and vertical components of its velocity.



Horizontal component:  $15 \cos 30^\circ = \mathbf{13 \text{ ms}^{-1}}$

Vertical component:  $15 \sin 30^\circ = \mathbf{7.5 \text{ ms}^{-1}}$

2. A football is kicked with a velocity of  $20 \text{ ms}^{-1}$  at an angle of  $60^\circ$  to the horizontal. Calculate the time of flight before the football hits the ground.

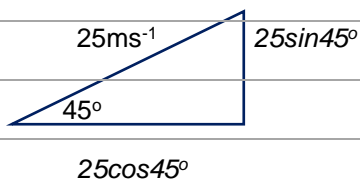


Vertical component:  $a = v - u / t$  so  $t = v - u / g$

$$t = 20 \sin 60^\circ / 9.81 = 1.77 \text{ seconds}$$

$$2 \times t = \mathbf{2.5 \text{ seconds}}$$

3. A projectile is launched at a speed of  $25 \text{ ms}^{-1}$  at an angle of  $45^\circ$ . Calculate the range of the projectile (assuming no air resistance).



Vertical component:

$$t = v - u / g$$

$$= 25 \sin 45^\circ / 9.81$$

$$= 1.8 \text{ s} \quad \text{so total time} = 3.6 \text{ s}$$

Horizontal component:

$$s = v t$$

$$= 25 \cos 45^\circ \times 3.6$$

$$= \mathbf{63.6 \text{ metres}}$$

4. A ball is thrown horizontally at  $10 \text{ ms}^{-1}$  from a cliff that is 20 meters high. How long does it take for the ball to reach the ground?

Vertical component only:  $s = ut + \frac{1}{2} at^2$

$$20 = 0 + \left( \frac{1}{2} \times 9.81 \times t^2 \right)$$

$$(20 \times 2) / 9.81 = t^2$$

$$\sqrt{4.077} = t = \mathbf{2.0 \text{ seconds}}$$

5. A rock is thrown vertically with a speed of  $18 \text{ ms}^{-1}$ . How high does it go before it starts falling back down?

Vertical component only:  $v^2 = u^2 + 2as$


$$0 = 18^2 + (2 \times (-9.81) \times s)$$

$$s = 18^2 / (2 \times 9.81) \quad \text{[negative signs cancel]}$$

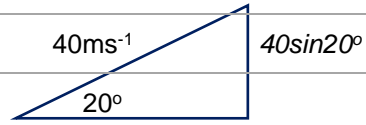
$$= \mathbf{16.5 \text{ metres}}$$

## Intermediate Questions

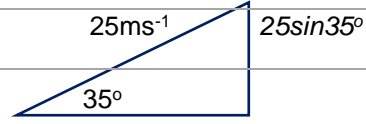
6. A projectile is launched at a speed of  $30 \text{ ms}^{-1}$  at an angle of  $37^\circ$  from the horizontal. Calculate both the total time of flight and the range.

	Vertical component:	Horizontal component:
	$t = v - u / g$	$s = v t$
	$= 30 \sin 37^\circ / 9.81 = 1.84$	$= 30 \cos 37^\circ \times 3.68$
	$2 \times t = 3.68 = \mathbf{3.7 \text{ seconds}}$	$= \mathbf{88 \text{ ms}^{-1}}$

7. A tennis ball is served with an initial velocity of  $40 \text{ ms}^{-1}$  at an angle of  $20^\circ$  to the horizontal. Calculate the maximum height reached by the ball.

	Vertical component:	$v^2 = u^2 + 2as$
		$0 = (40 \sin 20^\circ)^2 + (2 \times (-)9.81 \times s)$
		$187.2 / (2 \times 9.81) = s$
		$= \mathbf{9.5 \text{ metres}}$

8. A projectile is fired from the ground with an initial velocity of  $25 \text{ ms}^{-1}$  at an angle of  $35^\circ$ . If the projectile is in the air for 3.2 seconds, calculate its range.

	Horizontal component:	$s = v t$
		$= 25 \times \cos 35^\circ \times 3.2$
		$= 65.5 = \mathbf{66 \text{ metres}}$

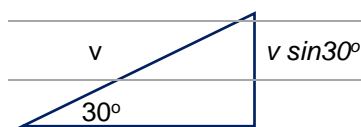
9. A projectile is fired horizontally from a height of 50 meters with an initial speed of  $20 \text{ ms}^{-1}$ . Calculate its speed just before it hits the ground.

Horizontal speed : $20 \text{ ms}^{-1}$			
To find vertical speed:	$v^2 = u^2 + 2as$	$= 0 + (2 \times 9.81 \times 50)$	$= 981$
	$v = \sqrt{981}$	$= 31.3 \text{ ms}^{-1}$	
Use Pythagoras to find total speed:	$\text{speed}^2 = 20^2 + 31.3^2 = 1381$	$\text{speed} = \sqrt{1381} = \mathbf{37 \text{ ms}^{-1}}$	



10. A projectile is launched at an angle of  $30^\circ$  and reaches a maximum height of 45 meters. What was its initial velocity?

Vertical component:  $v_y^2 = u_y^2 + 2as$



$$0 = u^2 + (2 \times (-)9.81 \times 45)$$

$$u = \sqrt{2 \times 9.81 \times 45} = 29.71 \text{ ms}^{-1}$$

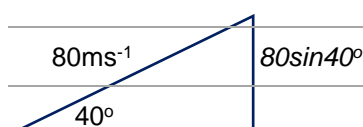
$$v \cos 30^\circ$$

so:  $v \sin 30^\circ = 29.71$        $v = 29.71 / \sin 30^\circ = \mathbf{59 \text{ ms}^{-1}}$

11. A cannonball is launched with an initial velocity of  $80 \text{ ms}^{-1}$  at an angle of  $40^\circ$ . Calculate both the time taken to reach its maximum height and the speed at that point.

Vertical component:

Horizontal component:



$$t = v - u / g$$

(there is no vertical speed at the apex)

$$= 80 \sin 40^\circ / 9.81$$

$$v = 80 \cos 40^\circ$$

$$80 \cos 40^\circ$$

$$= \mathbf{5.2(4) \text{ seconds}}$$

$$= \mathbf{61.(3) \text{ ms}^{-1}}$$

## Advanced Applications

12. An airplane flying horizontally at  $120 \text{ ms}^{-1}$  releases a package from a height of 500 m. How far horizontally from the point of release does the package land?

Find the time to fall:  $s = ut + \frac{1}{2} at^2$

$$500 = 0 + (\frac{1}{2} \times 9.81 \times t^2)$$

find the distance:  $s = v t$

$$t^2 = 500 / (\frac{1}{2} \times 9.81)$$

$$= 120 \times 10.1$$

$$t = \sqrt{101.94}$$

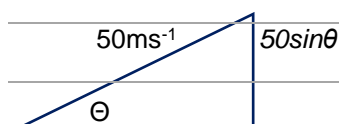
$$= 1212 \text{ m}$$

$$= 10.1 \text{ seconds}$$

$$= \mathbf{1.2 \text{ km}}$$

13. A projectile is launched with a speed of  $50 \text{ ms}^{-1}$ . At what angle should it be launched to achieve a range of 200 meters?

[we are going to work backwards – starting with the using the range to find the time]



Horizontal component:  $t = s / v = 200 / 50 \cos \theta$

Vertical component:  $\frac{1}{2} t = v - u / g$  so  $t = 2 \times 50 \sin \theta / 9.81$

$$50 \cos \theta$$

substituting for t:

$$\frac{200}{50 \cos \theta} = 2 \times \frac{50 \sin \theta}{9.81}$$

$$\text{so } \frac{200 \times 9.81}{50 \times 50} = 2 \sin \theta \cos \theta$$

[trig identity  $2 \sin \theta \cos \theta = \sin 2\theta$ ]

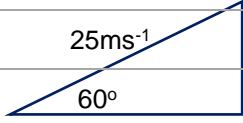
$$\frac{200 \times 9.81}{50 \times 50} = \sin 2\theta$$

$$2\theta = 51.702^\circ \quad \theta = \mathbf{26^\circ}$$



14. A ball is thrown from a height of 5 meters at an initial velocity of  $25 \text{ ms}^{-1}$  at an angle of  $60^\circ$ . Calculate its speed when it hits the ground.

Calculate time to reach apex:  $t = v - u / g = 25 \sin 60^\circ / 9.81 = 2.207 \text{ seconds}$

	<p>Calculate the total height reached: <math>s = ut + \frac{1}{2}at^2</math></p> <p><math>= (25 \sin 60^\circ \times 2.207) + (\frac{1}{2} \times 9.81 \times 2.207^2) = 47.78 + 10.83 = 58.61</math></p>
<p><math>25\cos 60^\circ</math></p>	<p>It falls 5 metres further so total drop is: <math>58.61 + 5 = 63.6 \text{ m}</math></p>

To find vertical speed as it hits the ground:  $v^2 = u^2 + 2as = 2 \times 9.81 \times 63.6 = 1248$

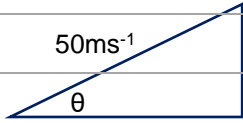
$v = \sqrt{1248} = 35.3 \text{ ms}^{-1}$

Using Pythagoras to find total speed:  $\text{speed}^2 = 35.3^2 + (25 \cos 60^\circ)^2$

Speed =  **$37 \text{ ms}^{-1}$**

15. A projectile is launched at  $50 \text{ ms}^{-1}$  from the ground and must hit a target 100 meters away at the same level. At what two angles can the projectile be launched to achieve this?

Time to target:  $t = s / v = 100 / 50 \cos \theta$

	<p>Time to apex: <math>\frac{1}{2}t = v - u / g</math> so <math>t = 2 \times 50 \sin \theta / 9.81</math></p> <p>Equating these: <math>\frac{100}{50 \cos \theta} = 2 \times \frac{50 \sin \theta}{9.81}</math> so: <math>\frac{100 \times 9.81}{50 \times 50} = 2 \sin \theta \cos \theta</math></p>
<p><math>50\cos \theta</math></p>	<p><math>[trig \text{ identity } 2 \sin \theta \cos \theta = \sin 2\theta]</math> <math>\frac{100 \times 9.81}{50 \times 50} = \sin 2\theta</math> <math>2\theta = 23.10^\circ \text{ or } 156.90^\circ</math></p>

**$\theta = 12^\circ \text{ or } 78^\circ$**

